Exercise 2

1. Find the hyperplanes specified by the directions $\boldsymbol{\xi}$ and points p as shown

   (a) $\boldsymbol{\xi} = (1, 0, 2); \quad \mathbf{p} = (0, 1, -4),$

   (b) $\boldsymbol{\xi} = (11, 3, 4, 0); \quad \mathbf{p} = (1, -1, -2, -5);$

   (c) $\boldsymbol{\xi} = (2, -1); \quad \mathbf{p} = (3, 3)$.

2. Find the plane passing through $(0, -1, 1)$ and parallel to the plane $x + y + 4z = 8$.

3. Find all planes passing through $(2, -3, 0)$ and intersecting the plane $x - 4y + 2z = -1$ vertically.

4. Find the hyperplanes passing through the following points:

   (a) $(1, 9), \ (2, -12)$,

   (b) $(1, 0, -1), \ (2, 0, 3), \ (1, 1, 0)$.

   (c) $(1, 1, 1), \ (1, -1, 1), \ (-1, 1, 1)$.

   (d) $(1, 1, 0, -4), \ (0, 0, 0, 3), \ (0, -1, 0, -1), \ (4, 2, 0, 0)$.

5. Show that the plane passing through $\mathbf{a}, \mathbf{b}$ and the origin is given by the determinant equation

$$\begin{vmatrix} x & y & z \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0 .$$

   What happens if $\mathbf{a}$ and $\mathbf{b}$ are collinear?

6. Find the distance from $\mathbf{P}$ to the hyperplane H and the point $\mathbf{Q}$ on H realizing the distance:

   (a) $(1, 2)$, $\quad x + 6y = 0$.

   (b) $(1, 2, 4)$, $\quad x - 2y = -1$,

   (c) $(2, 4, 6, 8)$, $\quad x - 3y - 4z + w = -6$.

7. Find two parametric forms of the straight lines described by the following system using x and z as the parameters respectively:

$$\begin{cases} 2x - 6y = 0, \\ 3x + 5y - 6z = -1 . \end{cases}$$

8. Find the point of intersection (if any) of the lines and planes:

   (a) $(0, 2) + t(1, 1)$, $\quad x + 3y = 1$.

   (b) $(7, 3, 0) + t(-4, 6, 5)$, $\quad 4x + y + 2z = 17$.

   (c) $(15, 10, 5) + t(7, 12, -4)$, $\quad x + y + z = 45$.

9. The angle between two planes is defined to be the angle between their normal directions (lying in $[0, \pi)$). Find these angles in the following cases:

   (a) $2x - y + z = 13$ and $x + y - z = 1$.

(b) $x + 11y - 5z = 0$ and the xy-plane.

10. Find the straight lines passing through $(1, -2, 3)$ and hitting the plane $x - y + 6z = -1$ at right angle. Find the point of intersection too.

11. Find the distance between the two straight lines in the following cases:

    (a) $(1, 2, 3) + t(0, 1, 0)$ and $s(1, 0, -2)$.

    (b) $(1, 0, 1, 0) + t(-1, 0, 0, 0)$ and $(-1, -1, 0, 1) + s(0, 2, -1, 0)$.

    Hint: The distance is realized by the line segment that is perpendicular to both lines.

12. Propose a definition of the projection of $P(x, y, z)$ on a straight line passing through the origin and then find a formula for it.

13. Find the three medians of the triangle $A(0, 0)$, $B(2, 6)$, $C(4, -4)$ and verify that they meet at a point.

14. Let $A(1, 0)$, $B(2, 3)$, $C(4, 4)$ be a triangle. Determine its altitude from A to BC and from B to AC.

15. * Let $A(3, 4)$, $B(0, 0)$, $C(2, 0)$ be a triangle. Determine its angle bisector from A and from C.

16. Find the "standard forms" of the following quadratic equations and describe their solution sets.

    (a) $x^2 - 2xy + 2y = 0$,

    (b) $x^2 + 2xy + y^2 + 2y = -6$,

    (c) * $5x^2 + 4y^2 - 2xy + ax = -1$, $\quad a \in \mathbb{R}$.